



IDENTIFYING THE CRITICAL FACTORS OF STUDENTS' DIFFICULTY UNDERSTANDING THE CONCEPT OF LOGARITHMS

Ariyani Muljo¹, Budi Irwansyah^{2*}, Wanda Darmawan³,
Juhendrifansyah⁴, Ira Keumalasari⁵

^{1,2} IAIN Langsa, Aceh, 24411, Indonesia

³ SMA Negeri 2 Percontohan Karang Baru, Aceh Tamiang

⁴ SMA Negeri 1 Kejuruan Muda, Aceh Tamiang

⁵ SMA Negeri 2 Kejuruan Muda, Aceh Tamiang

* Corresponding email: budi.irwansyah@iainlangsa.ac.id

ABSTRACT

Logarithms are one of the mathematics lesson materials in the algebra aspect which provide many benefits in everyday life. Despite the many benefits associated with logarithms, most students think that logarithms are something difficult and not worth learning. This research aims to determine important factors in students' difficulties in understanding the concept of logarithms. The samples were 99 class X students from three public high schools in Aceh Tamiang Regency. Data was obtained using a logarithm material diagnostic test instrument based on aspects of logarithms as an object, process, function, and contextual problems. From these four aspects, we identified 17 variables that caused students difficulty in understanding the concept of logarithms. The research results show that there are five main factors that cause students difficulties in understanding the concept of logarithms. These factors are: (a) connecting square roots with logarithms, (b) connecting logarithmic and exponential functions as inverses, (c) rules of logarithmic notation, (d) logarithmic graphs, and (e) simplifying logarithmic expressions.

Keywords: *factor analysis; students difficulties; concept understanding; logarithm.*

INTRODUCTION

One of the aims of studying mathematics was for students to be able to cultivate the capacity to think critically, logically, systematically, conscientiously, effectively, and efficiently in solving problems (BNSP,

2006). Mathematics is an important science, but in reality, mathematics lessons are less popular, feared, and boring for students (Novriani & Surya, 2017). As a result, several students have difficulty learning mathematics. Students who experience difficulties in learning mathematics and get bored quickly are students who have negative perceptions (Harahap & Syarifah, 2017). (Cooney, 1994) states that students' mathematics learning difficulties are classified into three types of difficulties with the following criteria: (1) students' difficulties in using mathematical concepts, (2) students' difficulties in using mathematical principles, and (3) students' difficulties in solving verbal problems. Every student cannot avoid difficulties in learning mathematics at school, generally, students experience different levels of difficulty in learning mathematics (Widodo, 2013). Students who experience difficulties have the opportunity to make mistakes in solving mathematics problems in each subject of learning (Untari, 2013). Logarithm which is an aspect of algebra is no exception to the benefits algebra offers (Ansah & Vos, 2016). It finds application in areas such as medicine (Kinsella, 2007), economics (Azad, 2012), etc. In despite the benefits associated with logarithms, according to (Wong, et al., 2002) that most students think logarithms are something that is difficult and has no use in everyday life. (Liang & Wood, 2005) also stated that the topic of logarithms is a difficult topic for students and there is a high prevalence of misconceptions in students' thinking. The difficulty can lie in one of the four mathematical objects, namely concepts. A concept is an abstract idea that allows someone to classify an object and explain whether the object is an example or not an example of an abstract idea. (Gamble, 2005) said that teachers often tell students that "logarithms are exponents", but for some reason students hear the terms "exponent" and "logarithm" but often do not understand the relationship between the two. (Berezovski & Zazkis, 2006) found that students had difficulty recognizing logarithms as numbers. They expressed doubt that problems in calculating logarithmic expressions involving only numbers using either a calculator or manual calculations interpret the understanding of logarithms as numbers. Students may have learned the procedure by which certain types of problems are presented but do not understand that logarithmic expressions are numbers. This is in line with research by (Kastberg, 2002) which reports that students who can solve problems do not necessarily perceive logarithmic expressions as numbers. (Hoon, Singh, & Ayop, 2010) in their research found that students were able to do routine calculations in logarithms, but were less able to solve problems that required high-level cognitive thinking. (Ganesan & Dindyal, 2014) stated that students' errors in logarithms can be related to their understanding of the topic which implies knowledge of their mathematical thinking and mental constructs. (Weber, 2016) focuses on logarithms as numbers or operators, where logarithms can be conceptualized as inverse exponents (i.e. structurally). Since the indirect definition is still valid in algebra textbooks despite the many problems

reported in the educational literature, it may be a major cause of student difficulties. (Dintarini, 2018) found that some students did not fully understand the meaning of logarithms. In the end, students had difficulty applying the definition of logarithms to problems. At first students seemed able to calculate the logarithm value of a number, but failed when faced with more complex logarithms. Meanwhile, (Rafi & Retnawati, 2018) found that the common mistakes students made in logarithmic material were technical errors, distorted theorems or definitions, unverified solutions, and data usage errors. (Liang & Wood, 2005) also found that students appeared to be able to perform routine calculations but were less able to answer questions that required a higher level of cognitive thinking. In addition, many errors are not caused by a lack of knowledge but appear to be due to an over-generalization of algebraic rules. Students tend to have low understanding regarding the initial concept of logarithmic functions, students have difficulty understanding the application of logarithmic functions, and difficulties in solving logarithmic functions. According to (Hayati & Budiyo, 2018), the possible causes of errors in logarithmic learning come from teachers, students, or learning tools. Student errors in completing several tasks related to logarithmic function material became an obstacle in the next learning process. According to (Sinaga, et al., 2014) students must be able to master the concepts and properties of logarithms before studying logarithmic functions. Then, what are the dominant factors that cause students to have difficulty understanding the concept of logarithms? This question is what makes the author interested in exploring students' difficulties in understanding logarithmic material at three senior high schools in Aceh Tamiang Regency.

METHODS

The location of this research was conducted at the SMA Negeri 2 Percontohan Karang Baru, SMA Negeri 1 Kejuruan Muda, and SMA Negeri 2 Kejuruan Muda, in Aceh Tamiang Regency. Ninety-nine class X students were taken as a sample. To obtain research data, Authors 3, 4, and 5 who is Mathematics teacher at the research location, used a diagnostic test instrument for logarithmic material based on the aspects proposed by (Williams, 2011), namely the logarithm is divided into four parts; logarithms as objects, as processes, as functions, and in contextual matters. Furthermore, from these four aspects, Authors 3, 4, and 5 compiled a questionnaire and succeeded in identifying 17-factor items that caused class X students to have difficulty understanding the concept of logarithms. The indicators can be seen in Table 1. A quantitative approach is used to analyze the collected data so that the factors can be identified. The Explanatory Factor Analysis (EFA) type factor analysis method was chosen to reduce the variables represented by the questionnaire constituent items and the Principal Components Analysis (PCA) technique was used to form the factors. According to (Johnson & Wichern,

2007), PCA is the simplest procedure for extracting factors. This method forms a linear combination of observed indicators.

Table 1. Indicators

No.	Variables
1	know the meaning of ${}^a\log b$
2	know that logarithmic expressions are numbers, and don't need to be approximated by decimals
3	flexibly change the form of logarithmic expressions using logarithm rules
4	know the rules of logarithmic notation and how they relate to the order of operations
5	knows that a logarithmic expression must have a base and an argument
6	simplifies logarithmic expressions
7	estimate the value of a logarithmic expression
8	use logarithms by eliminating the base of exponential expressions when solving exponential equations
9	uses logarithms to find the value of the exponent in an expression
10	Use logarithms in the repeated division problem
11	connecting square roots with logarithms
12	"insert" a value in the function domain that will produce a single value
13	know logarithm function graphs in general and understand how to make logarithm graphs
14	understand that logarithmic functions have a finite domain, and an infinite range
15	explains why there are vertical asymptotes, but no horizontal asymptotes, in logarithmic graphs
16	connect logarithmic and exponential functions as inverses
17	understand the constraints of solutions to equations and inequalities involving logarithms

RESULTS AND DISCUSSION

The data feasibility test is used to see whether the data in the questionnaire is suitable for processing using factor analysis. The data suitability test is determined by looking at the Bartlett test of sphericity, Kaiser Meyer-Olkin (KMO), and Measures of Sampling Adequacy (MSA) values that are sought for each indicator. The quantity values that must be met according to (Johnson & Wichern, 2007), the Bartlett test of sphericity value with significance < 0.05 , the KMO value must be ≥ 0.50 and the MSA value must be ≥ 0.50 .

Table 2. KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		0.766
Bartlett's Test of Sphericity	Approx. Chi-Square	532.566
	Df	136
	Sig.	.000

In Table 2, the Bartlett's test of sphericity value shows a sig (level of significance) of 0.000. The KMO Measure of Sampling Adequacy figure of 0.766 with a significance of 0.000. Because the KMO figure is > 0.5 and the significance is far below 0.05, the existing variables and samples can be analyzed further using factor analysis. The next step is to carry out the MSA test, according to (Johnson & Wichern, 2007) the MSA value must be ≥ 0.50 , in this study the MSA criteria used was > 0.60 . The MSA test for all variables can be seen in Table 3:

Table 3. MSA Value of All Indicators

Index	Variables	MSA value
X ₁	know the meaning of "log b	0,697
X ₂	know that logarithmic expressions are numbers, and don't need to be approximated by decimals	0,833
X ₃	flexibly change the form of logarithmic expressions using logarithm rules	0,637
X ₄	know the rules of logarithmic notation and how they relate to the order of operations	0,683
X ₅	knows that a logarithmic expression must have a base and an argument	0,834

X ₆	simplifies logarithmic expressions	0,788
X ₇	estimate the value of a logarithmic expression	0,763
X ₈	use logarithms by eliminating the base of exponential expressions when solving exponential equations	0,745
X ₉	uses logarithms to find the value of the exponent in an expression	0,782
X ₁₀	Use logarithms in the repeated division problem	0,862
X ₁₁	connecting square roots with logarithms	0,880
X ₁₂	"insert" a value in the function domain that will produce a single value	0,800
X ₁₃	know logarithm function graphs in general and understand how to make logarithm graphs	0,710
X ₁₄	understand that logarithmic functions have a finite domain, and an infinite range	0,691
X ₁₅	explains why there are vertical asymptotes, but no horizontal asymptotes, in logarithmic graphs	0,559
X ₁₆	connect logarithmic and exponential functions as inverses	0,735
X ₁₇	understand the constraints of solutions to equations and inequalities involving logarithms	0,775

From Table 3, it can be seen that all variables have an MSA number > 0.5 ; this means that variables can still be predicted and analyzed further using factor analysis and none of the variables must be eliminated. Factors with an eigenvalue of more than 1 are the factors that will be selected. The greater the eigenvalue of a factor, the better the factor is at representing a number of variables. By using the Principal Component Analysis (PCA) method, factors with an eigenvalue of more than 1 can be seen in Table 4.

Table 4. Number of Factors Formed

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %

1	5.039	29.641	29.641	5.039	29.641	29.641	3.496	20.563	20.563
2	1.753	10.312	39.953	1.753	10.312	39.953	2.378	13.988	34.551
3	1.473	8.664	48.617	1.473	8.664	48.617	2.032	11.956	46.507
4	1.358	7.988	56.605	1.358	7.988	56.605	1.504	8.848	55.355
5	1.039	6.110	62.715	1.039	6.110	62.715	1.251	7.360	62.715
6	0.985	5.795	68.510						
7	0.760	4.468	72.978						
8	0.729	4.287	77.265						
9	0.672	3.953	81.218						
10	0.571	3.359	84.577						
11	0.534	3.141	87.718						
12	0.462	2.719	90.437						
13	0.432	2.538	92.975						
14	0.353	2.079	95.055						
15	0.344	2.021	97.075						
16	0.294	1.730	98.805						
17	0.203	1.195	100.000						

Table 5 shows that the number of factors formed is five factors, namely the first factor has an eigenvalues number of 5.039, the second factor is 1.753, the third factor is 1.473, the fourth factor is 1.358, and the fifth factor is 1.039, but for the sixth factor the eigenvalues number is already < 1 , namely 0.985, so the factor analysis process stops at the fifth factor only. The total percentage variance of the five factors is $29.641\% + 10.312\% + 8.664\% + 7.988\% + 6.110\% = 62.715\%$. This means that 62.715% of all existing variables can be explained by the five factors formed. In this study, factor analysis with Principal Component Analysis (PCA) extraction obtained five factors that cause difficulties in understanding the concept of logarithms. These factors have a total of seventeen variables. The five factors in the PCA

extraction method are each given a name according to the grouping of results on factor rotation. The results of the varimax factor rotation can be seen in Table 5:

Table 5. Results of Factor Rotation

	Component				
	1	2	3	4	5
1	.407	.645	-.105	-.345	.086
2	.704	.265	.007	-.118	.058
3	.193	-.057	.705	-.302	-.063
4	.136	.026	.722	.089	.263
5	.703	.156	.189	-.179	-.035
6	.143	.203	.144	.060	.701
7	.511	.057	.064	.253	-.506
8	.550	-.103	.458	.276	.314
9	.667	.081	.310	.237	.113
10	.659	.006	.092	.181	.511
11	.715	.183	.050	.267	.035
12	.315	.527	.060	-.037	-.090
13	.034	.381	.709	.138	-.038
14	.097	.607	-.065	.432	.147
15	.122	.167	.006	.823	.006
16	.098	.710	.046	.226	.118
17	.098	.667	.337	.039	.014

From Table 6, the results of the matrix component rotation process show the distribution of variables for each factor more clearly and realistically. This

process can clarify the position of a variable on a factor. Seventeen variables that have been reduced to form five factors. Next, based on Table 4, it can be seen that there are seventeen variables spread across five factors with a total variance of 62.715%. Based on the results of the varimax rotation, it can be seen that all variables form five factors. Each of the five factors is given a new name according to the grouping of variables in that factor. However, it should be noted that the factor naming process is not very scientific and is based on the subjective opinion of the analyst (Hair, Black, Babin, & Anderson, 2010).

a. Factor 1 consists of 7 elements : X_2 (know that logarithmic expressions are numbers, and don't need to be approximated by decimals), X_5 (knows that a logarithmic expression must have a base and an argument), X_7 (estimate the value of a logarithmic expression), X_8 (use logarithms by eliminating the base of exponential expressions when solving exponential equations), X_9 (uses logarithms to find the value of the exponent in an expression), X_{10} (Use logarithms in the repeated division problem), X_{11} (connecting square roots with logarithms). Factor 1 can be named "connecting square roots with logarithms". This is in line with the research (DePierro, Garafalo, & Toomey, 2008), which stated that first-year and advanced students in high education need to be helped to overcome difficulties in understanding the concept of logarithms by reviewing one of the basic math skills, namely square roots. These factors has a variance of 29.641%. This means that 29.641% of the difficulties in understanding the concept of logarithms are determined by the connecting square roots with logarithms.

b. Factor 2 consists of 5 elements: X_1 (know the meaning of a log b), X_{12} ("insert" a value in the function domain that will produce a single value), X_{14} (understand that logarithmic functions have a finite domain, and an infinite range), X_{16} (connect logarithmic and exponential functions as inverses), and X_{17} (understand the constraints of solutions to equations and inequalities involving logarithms). Factor 2 can be named "connect logarithmic and exponential functions as inverses". This is in line with research, namely (Gómez-Chacón & Joglar, 2010) which stated that the learning of these functions causes serious difficulties for students. These factors have a variance of 10.312%. This means that 10.312% of the difficulties in understanding the concept of logarithms are determined by connecting logarithmic and exponential functions as inverses.

c. Factor 3 consists of 3 elements: X_3 (flexibly change the form of logarithmic expressions using logarithm rules), X_4 (know the rules of logarithmic notation and how they relate to the order of operations), X_{13} (know logarithm function graphs in general and understand how to make logarithm graphs). Factor 3 can be named "rules of logarithmic notation". This is in line with the previous research literature, namely (Berezovski &

Zazkis, 2006) which stated that students' strategies for overcoming logarithm questions still depend on rules. Supported by other research, namely (Hurwitz, 1999) which stated that students often have difficulty thinking of logarithms as the output of a function because the notation used for logarithms does not look like the usual $f(x)$ notation. These factors have a variance of 8.664%. This means that 8.664% of the difficulties in understanding the concept of logarithms are determined by the rules of logarithmic notation.

d. Factor 4: X_{15} (explains why there are vertical asymptotes, but no horizontal asymptotes, in logarithmic graphs). Factor 4 can be named "logarithmic graphs". This is in line with the research (Aziz, Pramudiani, & Purnomo, 2017), which stated that when students sketch logarithm graphs, they assume the graph is symmetrical to certain quadrants and other quadrants, but that is an overgeneralization that leads to errors. This factor has a variance of 7.988%. This means that logarithmic graphs determine 7.988% of the difficulties in understanding the concept of logarithms.

e. Factor 5: X_6 (simplifying logarithmic expressions). Factor 5 can be named "logarithmic expressions". This is in line with the previous research literature, namely (Kastberg, 2002), which stated that students who can solve problems do not necessarily perceive logarithmic expressions as numbers. This factor has a variance of 6.110%. This means that simplifying logarithmic expressions determines 6.110% of the difficulties in understanding the concept of logarithms.

Table 6. Transformation Matrix Components

Component	1	2	3	4	5
1	.752	.471	.370	.200	.188
2	-.150	.701	-.644	.233	-.134
3	-.630	.310	.545	.343	.306
4	-.123	.438	.255	.822	-.231
5	-.003	-.013	.295	.337	.894

From Table 6, it can be seen that the diagonal between components 1 and 1, components 2 and 2, components 3 and 3, components 4 and 4, and components 5 and 5, the five numbers are visible (0.752; 0.701; 0.545; 0.822 and 0.894) above 0.5. This proves that the five factors (components) formed are correct, because they have a high correlation between the factors before rotation and the factors after rotation.

CONCLUSION

The research results show that several factors determine the difficulties in understanding the concept of logarithms. These factors are connecting square roots with logarithms, connecting logarithmic and exponential functions as

inverses, rules of logarithmic notation, logarithmic graphs, and simplifying logarithmic expressions. These factors were obtained from exploratory tests using the factor analysis method. We defined constructs based on relevant literature, namely strong previous research. The aim of looking for the difficulties in understanding the concept of logarithms is to obtain a model that can be used by teachers as a reference in mathematics learning. Not only in terms of students but also in other aspects of the school as a whole. The strategy to improve the quality of mathematics learning also aims to face competition from other schools that focus on students. The results of research using exploratory factor analysis (EFA) also provide the fact that there are other important factors as components of difficulties in understanding the concept of logarithms, which are not included in this research. Future EFA studies should continue to observe recommended guidelines by reporting the type of extraction and rotation methods used, and the methods used to retain factors and by using multiple methods (e.g., extraction, rotation, factor retention). The reporting of complete details about all the choices made when conducting an EFA is strongly recommended. Most importantly, researchers need to be aware of the proper use of PCA and common factor analysis and the potential negative consequences of relying on PCA with varimax rotation in the wrong research context.

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